Matlab
微積分應用

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(分機: 5715)
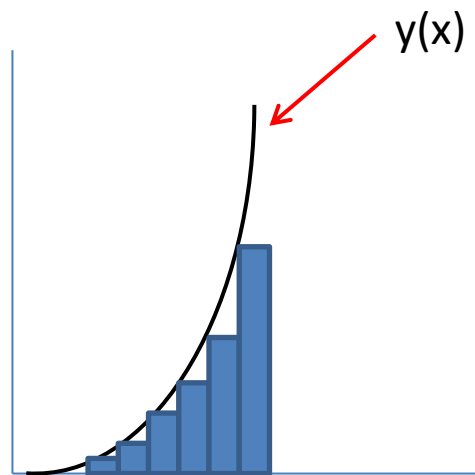
應用3-1: 求函數積分

- 不知函數f(x)，已知數據點：
- trapz:梯形積分法
- 已知函數f(x)：
- quad: 適應性辛普森法

y(x)

梯形積分法：trapz

基本積分表

$$\int k dx = kx + C$$

$$\int a x^n dx = a \frac{1}{n+1} x^{n+1} + C \quad (n \neq -1)$$

$$\int \cos x dx = \sin x + C$$

$$\int \sin x dx = -\cos x + C$$

$$\int e^x dx = e^x + C$$

$$\int \frac{k}{x} dx = k \ln|x| + C$$
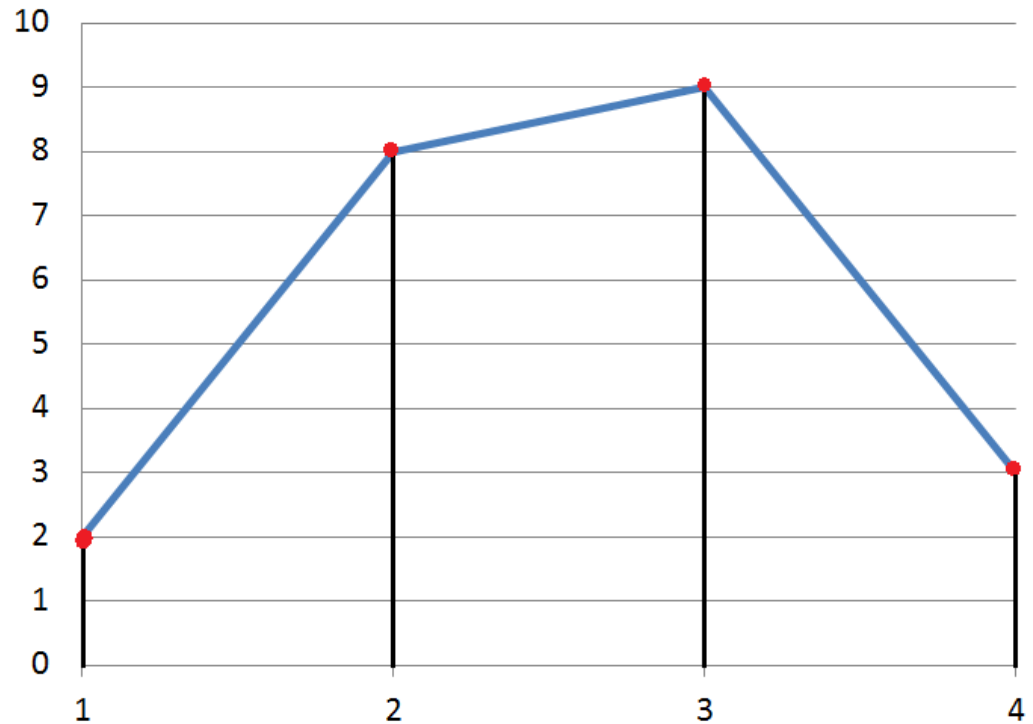
數值積分

- 梯形積分法

    A = trapz(x,y)

    – Ex:

    \>\> x = [1 2 3 4];

    \>\> y = [2 8 9 3];

    \>\> area = trapz(x,y)

    area = 19.5

數值積分

- 已知函數積分：quad

  A = quad('func',a,b)

  Func: 函式
  a: 積分下限
  b: 積分上限

  Ex1: $\int_{0}^{2} \dfrac{1}{x^3 - 2x - 5}\, dx$

  >> area = quad('1./(x.^3-2*x-5)',0,2)
  area = -0.4605

  Ex2: $\int_{1}^{2} exp(2x)dx$

  >> A = quad('exp(2*x)',1,2)
  A = 23.605

  NOTE:
  函數內之數學運算必須使用向量個別元素之運算
  (.* ./ .^)

應用3-2: 求函數微分

基本微分表
$\dfrac{\mathrm{d}}{\mathrm{d}x}kx^n = knx^{n-1}$
$\dfrac{\mathrm{d}}{\mathrm{d}x}a^x = a^x \ln a$
$\dfrac{\mathrm{d}}{\mathrm{d}x}e^x = e^x$
$\dfrac{\mathrm{d}}{\mathrm{d}x}\ln x = \dfrac{1}{x}$
$\dfrac{\mathrm{d}y}{\mathrm{d}t} = \dfrac{\mathrm{d}y}{\mathrm{d}x} \times \dfrac{\mathrm{d}x}{\mathrm{d}t}$

$$\frac{\mathrm{d}}{\mathrm{d}t}(6\sin(4t)) = 6 \times \frac{\mathrm{d}}{\mathrm{d}t}(\sin(4t))$$

$$= 6 \times \cos(4t) \times \frac{\mathrm{d}}{\mathrm{d}t}(4t)$$

$$= 6 \times \cos(4t) \times 4$$

$$= 24\cos(4t)$$

數值微分

- 已知數據點的微分
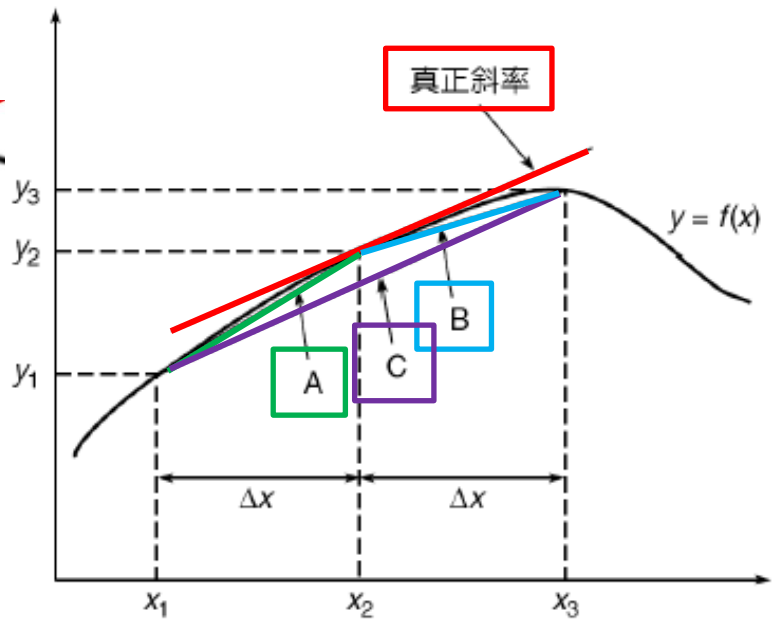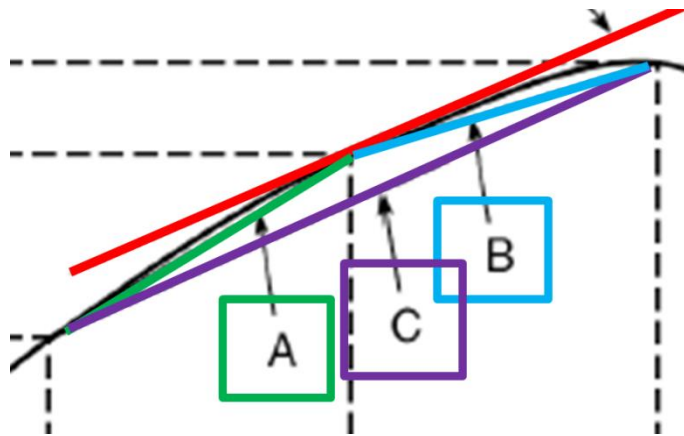
- 在 x_2 之微分

$$\text{Definition of Derivative: } \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

$$\text{Backward Difference: } m_A = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_2 - y_1}{\Delta x}$$

$$\text{Forward Difference: } m_B = \frac{y_3 - y_2}{x_3 - x_2} = \frac{y_3 - y_2}{\Delta x}$$

$$\text{Central Difference: } m_C = \frac{1}{2}\left(\frac{y_3 - y_2}{\Delta x} + \frac{y_2 - y_1}{\Delta x}\right) = \frac{y_3 - y_1}{2\Delta x}$$

Ex: 6sin(4t):

```
>> x = linspace(0,pi,40);
>> y = 6*sin(4*x);
>> d = diff(y)./diff(x); % backward or forward difference
>> dc = (y(3:end)-y(1:end-2))./(x(3:end)-x(1:end-2)); % central difference
>> dy = 24*cos(4*x); % 實際微分值
>> plot(x, dy, x(2:end), d,'o', x(1:end-1), d,'x', x(2:end-1), dc,'^')
>> xlabel('x','Fontsize',[60]);
>> ylabel('Derivative','Fontsize',[60])
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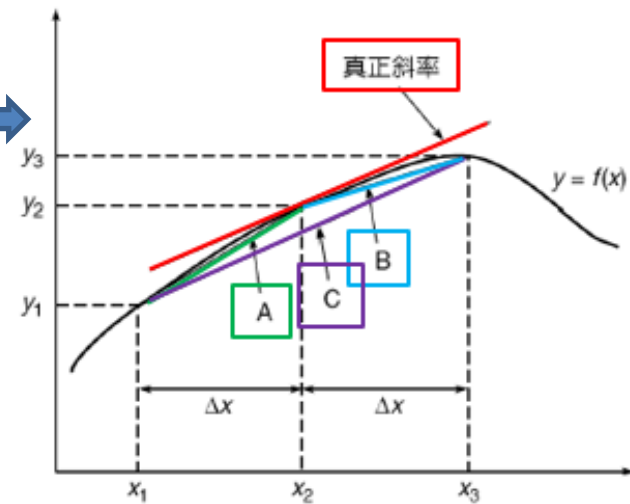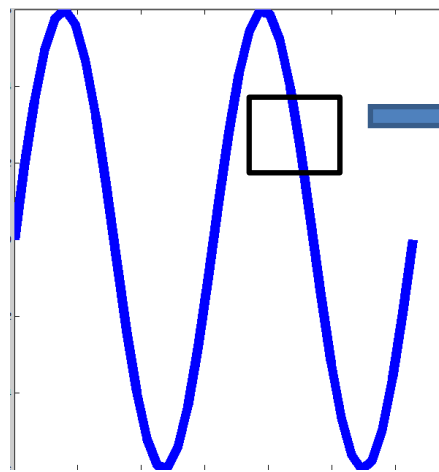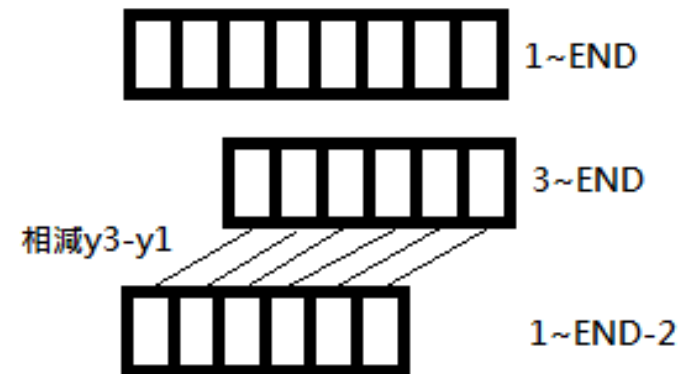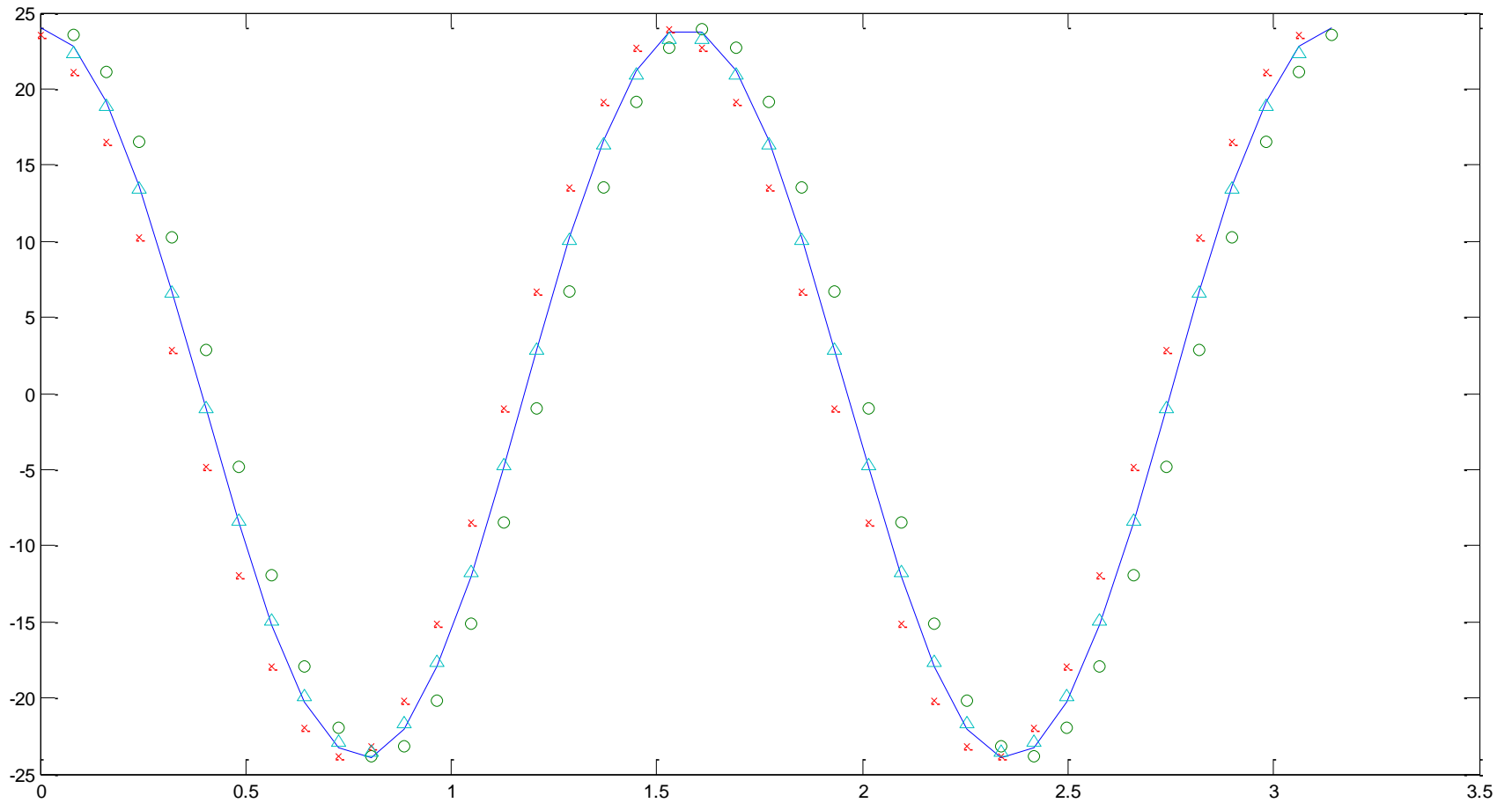
真正斜率

y_3

y_2

y_1

$y = f(x)$

B

A   C

Δx Δx

x_1 x_2 x_3

$$d = \text{diff}(x)$$
$$= [x(2) - x(1), x(3) - x(2), \ldots, x(n) - x(n-1)]$$

1~END

3~END

相減 y3-y1

1~END-2

O = backward difference

X = forward difference

△ = central difference

-- = 24*cos(4*x)